Relativistic Engineering

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Pascal Lecture Plan
(tentative, need your feedback)

• Nov.18: Second Lecture “Laser Electron Acceleration and its Future”
• Dec.9: Third Lecture “Laser Ion Acceleration”
• January, 20, 2010: “Relativistic Engineering”
• February, 26: “High Field Science”
• March: “Photonuclear Physics”
• April: “Medical Applications”
• ......
1000-fold Challenges
Frontier science driven by advanced accelerator

Can we meet the challenge? How can we meet it?

compact, ultrastrong a atto-, zeptosecond

(KEK: A. Suzuki)
\begin{align*}
\text{Density cusp} & \rightarrow \\
\text{Relativistic coherece} & \leftarrow \nu \text{ tends to } c; \text{ velocities condense toward } c \\
& \text{(cf. quantum coherence; energies condense toward 0)}
\end{align*}

\text{Condition for density cusp formation:}
\[ J = |\frac{\partial x}{\partial x_0}| \to 0 \]
\text{(\(\delta\)Eulerian coordinate/\(\delta\)Lagrangian coordinate) \(\to 0\)}

Cusp density happens: \[ \nu_e \rightarrow \nu_{ph} \sim c \]

Density diverges at wavehead
\[ n_e \bigg|_{x=x_{br}} \to \infty \]
RE / Flying Mirror Concept:


And more

In relativistic regimes, matter coheres more due to the consequence of relativity. (cf. quantum coherence) → ‘relativistic engineering’
Progress in Laser Power

- **Relativistic Engineering**
- **Nonlinear Optics in vacua**
- **Vacuum breakdown**

- **Schwinger limit**
  - Nonlinear QED: $E \cdot e \cdot \lambda \gamma = 2m_e c^2$
  - Schwinger limit

- **Zettawatt Laser**
  - Laser Intensity Limit: $I = \frac{h \nu^3}{c^2} \cdot \frac{\Delta \nu}{\sigma} = \frac{P_{th}}{\lambda}$
  - (“Mourou limit”)
  - Relativistic Optics: $v_{av} \sim c$
    - (large ponderomotive pressures)

- **Bound Electrons**
  - Bound Electrons: $E = \frac{e^2}{a_o}$
  - Nonlinear Optics (Bloembergen)

- **High Field Science**
  - CPA
  - Q-switching
  - Mode-locking

- **Progress in Laser Power**
  - MeV
  - PeV
  - TeV

- **Timeline**
  - 1960 to 2010

- **Photonuclear physics**
Flying Mirror
for Femto-, Atto-second, …
Science

(KEK: A. Suzuki)
Flying Mirrors

- Laser Light backscattered from coherent relativistic ‘flying mirror’:
  - frequency up-shifting [X-rays]
  - pulse compression [attosecond]
  - directed
  - coherent
  - single frequency (cf. HHG)
  - intense (cf. HHG -- Corkum limit)

Duality Conjecture: “Intensity and shortness of pulse go hand in hand.”
Relativistic flying mirrors are an example of this.
EM Pulse Intensification and Shortening by the Flying Mirror

3D Particle-In-Cell Simulation

(Bulanov, Esirkepov, Tajima, 2003)
Space-Time Overlapping of Driver and Source pulses

Side View

Vacuum focus

Jet center

\( t_p = 0 \)

Driver

200 mJ, 76 fs

Source

12 mJ

Top View

Relativistic Microlens

(Kando et al 2007)
Signals of backreflected photons off flying mirror at \( c \ (p > 0.5) \)

Estimated reflected photon number/sr

\[ N_x, \text{ 1/sr} \]

\[ p \]

\[ 1 \]

\[ 0.9 \]

\[ 0.8 \]

\[ 0.7 \]

\[ 0.6 \]

\[ 0.5 \]

\[ \lambda, \text{ nm} \]

\[ 6 \]

\[ 8 \]

\[ 10 \]

\[ 12 \]

\[ 14 \]

\[ 1 \times 10^7 \]

\[ 2 \times 10^7 \]

\[ 3 \times 10^7 \]

\[ 4 \times 10^7 \]

\[ \Delta \rho, \mu \text{m} \]

\[ 0 \]

\[ 10 \]

\[ 20 \]

\[ 30 \]

\[ 40 \]

\[ \mu \text{m} \]

\[ 12 \mu \text{m} \]

\[ 180 \text{ fs} \]

\[ \rho, \text{ arb. u.} \]

\[ 0 \]

\[ 0.25 \]

\[ 0.5 \]

\[ 0.75 \]

\[ 1.0 \]

\[ \lambda_x = 14.3 \text{ nm} \]

\[ \Delta \lambda_x = 0.3 \text{ nm}, \quad \Delta \lambda_x / \lambda_x = 0.02 \]

\[ \text{Wake wave parameters: } \gamma = 4.1, \quad \Delta \gamma / \gamma = 0.01 \]

\[ \sim 4 \times 10^7 \text{ photons/sr} \]

\[ \text{Reflected pulse duration: } \tau_x \sim 1.4 \text{ fs (femtosecond pulse)} \]

(Kando et al, 2007)
Laser pulse plasmawake wave

paraboloidal relativistic flying mirrors

Reflectivity

\[ R \approx \frac{1}{2\gamma} \]

Driver pulse: $a=1.7$
size=$3\lambda \times 6\lambda \times 6\lambda$, Gaussian
$I_{\text{peak}} = 4 \times 10^{18} \text{W/cm}^2 \times (1\mu\text{m}/\lambda)^2$

Source pulse:
$a=0.05$
size=$6\lambda \times 6\lambda \times 6\lambda$, Gaussian, $\lambda_s = 2\lambda$
$I_{\text{peak}} = 3.4 \times 10^{15} \text{W/cm}^2 \times (1\mu\text{m}/\lambda)^2$

3D PIC $t = 1.00$

(XZ, color: $E_y$
XY, contour: $E_z$
XY, color: $E_x$ at $z = 0$)

(Esirkepov)
Oblique incidence

\[ \beta = \frac{V}{c}; \]
\[ \gamma = \frac{1}{\sqrt{1 - \beta^2}} \]

\[ \omega_x \approx 4\gamma^2 \cos^2 \frac{\theta}{2} \omega_0 \]
\[ \tau_x \approx \frac{\tau_0}{4\gamma^2 \cos^2 \frac{\theta}{2}} \]
\[ \theta_x \approx \frac{1}{2\gamma^2} \tan \frac{\theta}{2} \]

Example: \( \theta = 45^\circ \)
\[ \omega_x \approx 3.4\gamma^2 \omega_0; \quad \tau_x \approx \frac{\tau_0}{3.4\gamma^2}; \quad \theta_x \approx \frac{1}{4.8\gamma^2} \]
## Compact Coherent Ultrafast X-Ray Sources

<table>
<thead>
<tr>
<th>X-ray source</th>
<th>Wavelength</th>
<th>Pulse Duration</th>
<th>Pulse Energy</th>
<th>Monochromaticity</th>
<th>Coherence</th>
</tr>
</thead>
<tbody>
<tr>
<td>XFEL (DESY)</td>
<td>13.8 nm</td>
<td>50 fs</td>
<td>100 μJ</td>
<td>10^{-3}</td>
<td>spatial good</td>
</tr>
<tr>
<td>Plasma XRL</td>
<td>13.9 nm</td>
<td>7 ps</td>
<td>1 μJ</td>
<td>10^{-4}</td>
<td>spatial good</td>
</tr>
<tr>
<td>Laser plasma</td>
<td>wide spectrum 1 nm – 40 nm</td>
<td>10 ps – 1 ns</td>
<td>10 μJ</td>
<td>1</td>
<td>no</td>
</tr>
<tr>
<td>HHG</td>
<td>30 – 40 nm</td>
<td>fs</td>
<td>1 μJ</td>
<td>10^{-2} – 10^{-3}</td>
<td>spatial good</td>
</tr>
<tr>
<td>Flying Mirror</td>
<td>0.1 – 20 nm</td>
<td>attosec</td>
<td>mJ</td>
<td>10^{-2} – 10^{-4}</td>
<td>spatial and temporal good</td>
</tr>
</tbody>
</table>

Predicted by the FM theory parameters of the x-ray pulse compared with the parameters of power x-ray generated by other sources
Moving mirrors in quantum electrodynamics

Dynamical Casimir effect


Unruh radiation


Unruh effect: accelerating observer sees black-body radiation.
Double-Sided Mirror

Wave loses energy
\[
\Delta E_{\text{loss}} = \hbar \omega - \hbar \omega \approx \left(1 - \frac{1}{4\gamma^2}\right) \hbar \omega
\]
(energy is transferred to the mirror)

Wave gains energy
\[
\Delta E_{\text{gain}} = \hbar \tilde{\omega} - \hbar \omega \approx (4\gamma^2 - 1) \hbar \omega
\]
(energy is taken from the mirror)

Radiation Pressure Dominant Acceleration (Laser Piston)

Flying Mirror


Accelerating Double-Sided Mirror

\[ V(t) \]

Driver \( \lambda_d \)

\[ \tilde{\lambda}_d \approx 4\gamma^2 \lambda_d \]

Source \( \lambda \)

\[ \tilde{\lambda} \approx \lambda/4\gamma^2 \]

\[ \tilde{\lambda}/n \approx \lambda/(n \times 4\gamma^2) \]

Kagami ("mirror" in Japanese)
Accelerating Double-Sided Mirror (Kagami)

Accelerating harmonics

Field along $x$-axis

\[ I_\omega(t) = \int_{-\infty}^{+\infty} E_z(\tau)e^{-i\omega \tau - c^2(\tau - t)^2/\lambda^2} d\tau \]

Spectrum at fixed time

Dashed curves:

\[ \frac{1 + \beta(\tau)}{1 - \beta(\tau)} \omega_0 \times (2n - 1), \quad n = 1, 2, 3, \ldots \]

$\tau$ – time of emission

time of detection:

\[ t = \tau - \int_0^\tau \beta(\tau)d\tau \]

Reflected light structure:

- Fundamental mode $\times 4\gamma^2$
- High harmonics $\times 4\gamma^2$
- Shift due to acceleration

(Esirkepov)
Interaction of an electromagnetic wave with an infinitely thin plasma slab

We seek the solution in the form:

\[ A(\xi, \eta) = \begin{cases} a_1(\xi) + a_0 e^{2i\eta}, & \psi(\xi, \eta) > 0; \\
                   a_2(\eta), & \psi(\xi, \eta) \leq 0. 
\end{cases} \]

\[ e^{2i\eta} = e^{ik(x+ct)} \]

Maxwell equation in terms of \( \xi, \eta \) reduces to ordinary differential equations:

\[
a'_1(\xi) = \chi \left( a_1(\xi) + a_0 e^{2i\eta_0(\xi)} \right) F_M(\xi, \eta_0(\xi)),
\]

\[
2ia_0 e^{2i\eta} - a'_2(\eta) = \frac{\chi}{F_M(\eta_0(\eta), \eta)} a_2(\eta) .
\]

\[
F_M(\xi, \eta) = \left[ \frac{1 + X'_M(\eta - \xi)}{1 - X'_M(\eta - \xi)} \right]^{1/2} \approx 2\gamma_M
\]

Uniformly accelerating mirror

Acceleration: \( g c^2 \)

\[ X_M(t) = g^{-1} \left\{ 1 + (g \bar{t})^2 \right\}^{1/2} \]

Solution

Reflected wave:

\[ a_1(\xi) = \frac{\chi a_0}{2g} \left( 2ig^2 \xi \right)^{\frac{\xi}{2g}} \Gamma \left( \frac{\chi}{2g}, \frac{1}{2g^2 \xi}, 0 \right) \]

Transmitted wave:

\[ a_2(\eta) = \frac{\chi a_0}{2g} \left( -2i\eta \right)^{-\frac{\chi}{2g}} \Gamma \left( \frac{\chi}{2g}, -2i\eta, 0 \right) + a_0 e^{2i\eta} \]

\[ \Gamma(a, z_1, z_2) = \int_{z_1}^{z_2} t^{a-1} e^{-t} dt \]

\[ a_1(\xi) = -\frac{\chi a_0}{2g} \left( 2ig^2 \xi \right)^{\frac{\xi}{2g}} \Gamma \left( \frac{\chi}{2g} \right) + i\chi a_0 g \exp \left( \frac{i}{2g^2 \xi} \right) (\xi + O(\xi^2)) \]

*Cf.* Reflection of plane waves from a uniformly accelerating mirror

Am. J. Phys. 69 (7), July 2001
Moving mirrors in classical electrodynamics

Relativistic Doppler effect
...

Free Electron Laser
...

Moving optical inhomogeneity
...
Quivering mirror

\[
\frac{d}{dt} \left( \frac{V(\tilde{t})}{\sqrt{1 - V^2(\tilde{t})}} \right) = g \cos(\Omega \tilde{t})
\]

\[
X_M(\tilde{t}) = \frac{1}{\Omega} \arctan \left( -\frac{\cos(\Omega \tilde{t})}{\sqrt{\Omega^2/g^2 + \sin^2(\Omega \tilde{t})}} \right)
\]

Solution

Reflected wave:

\[
a_1(\xi) = \frac{\chi a_0}{g} \int_{\xi}^{+\infty} \frac{E(\Omega \xi)}{E(\tau)} \frac{e^{-2i\tau}}{(h - ie^{2i\tau})^{\frac{3}{2}}} d\tau
\]

\[
E(\tau) = \exp \left\{ \frac{\chi}{g(h+1)} \text{F} \left( \tau - \frac{\pi}{4} \left| \frac{4h}{(h+1)^2} \right. \right) \right\}.
\]

\text{F}(z|m) - elliptic integral of the 1st kind.
The short pulse formation occurs whenever relativistically strong laser pulses interact with near-critical or overcritical plasmas, but, more responsive plasmas act more efficiently.

(N. Naumova et al)
Overview over SHHG Data from Berlin and first (preliminary) Interpretations

R. Hoerlein, D. has, S. Steinke, W. Sandner et al.
The CWE Mechanism

Glass Target (Density $\geq 2.6$ g/cm$^3$):

Plexiglass Target (Density $\leq 1.3$ g/cm$^3$):


... the only similar experiment ...

![Diagram]

**Harmonic Emission from the Rear Side of Thin Overdense Foils Irradiated with Intense Ultrashort Laser Pulses**


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(Received 25 November 2003; published 4 May 2004)

- foils 50nm to 400nm
- $a_0=0.5$ (at 2 omega)
- oblique incidence
- p-polarization
- harmonics visible from front and rear side
- density dependent cutoff in simulation
- experiment showed no signal for normal incidence
Odd – Even Asymmetry

unknown origin

• odd harmonics also exhibit cutoff (thus probably not relativistic)
• polarization of CWE not well studied (if at all!)

Possible reasons for observation

Odd generated more efficiently?

Polarization of odd and even harmonics different?

• strong influence of vxB? Not probable as circular polarization also shows effect…
• Different polarization of add and even? But why… anybody got ideas?

My newest idea…

• perhaps E-field effect but symmetry due to normal incidence?
• interference of two harmonic sources Pi out of phase left and right of center of focus???
Electron ejection can be achieved with large angles of incidence

A tightly focused laser pulse pushes plasma electrons inwards, creating:

- peaked electron density distribution,
- counter-streaming electrons,
- regions with minimal pressure (nulls), through which electrons jet.

Electron ejection is synchronized with attosecond pulse generation

Escaped relativistic electrons

- compress the reflected radiation into attosecond pulses and
- inherit a peaked density distribution.
- Complete modulation of e.m. field occurs. This is relativistic microelectronics!

Efficiency of attosecond phenomena: ~15% converted to attosecond pulses, ~15% to electron bunches.

...short electron bunches scatter coherently

- For counter-propagating light coherent scattering occurs with near unity efficiency when the bunch is short enough:
  \[ kd \approx \frac{1}{2\gamma^2} \]
- Then, \(10^8\) electrons are sufficient to reflect in \(1\mu m^2\)

Further investigation of the bunch characteristics and spectral content of the scattered radiation are needed

How to Produce Short Bunches

plasma acceleration!

(AS. Ogata)
Creation of a short bunch.

1 Use of high-density plasma with small $a_0$.

2 Bunch compression
sacrificing the energy width, to which the radiolysis is generous.

This talk is mainly on the 1st term.

N.E. Andreev, et al.,
LWFA Linear model says
fwhm bunch length < plasma wavelength/4

Different in nonlinear waves.
In nonlinear waves, \( \omega_p \) decreases

\[
\omega_p = \frac{\omega_{p0}}{\sqrt{1 + a_0^2}}
\]

P. Sprangle et al., PRL 64 (1990) 2011.
Paradigm shift by atto- and zepto-seconds:

- Measurement of electron wavefunction modulus squared $|\Psi|^2 \Rightarrow \Psi$
  
  - According to textbooks of QM, this quantity is measureable, by such methods as X-ray inelastic scattering.

- Measurement of electron wavefunction itself $\Psi$
  
  - According to textbooks of QM, this quantity is unmeasurable.
  
  $\Psi$: arbitrary phase with each electron. Phase of an individual electron in an as range
  
  $\Rightarrow$ origin of non-measurability of $\Psi$

- Emergence of coherent attosecond X-rays (relativistic engineering etc)
  
  - First ever possibility of electron wavefunction itself $\Psi$ (or its phase).

- Outstanding problems of contemporary physics:
  
  - Behavior of strongly-coupled systems
    
    (quantum coherent state due to electron many-body interaction
    
    e.g.) high Tc superconductors: 10nm coherent length

- New paradigm of matter control
  
  - Quantum control including the phase of matter
  
  - Departure from control philosophy in 1D energy domain $(\omega)$
  
  - Toward multidimensional control philosophy including phase $(\Phi)$

- Can we observe entangled quantum states? --- new question
Example of wavefunction of a Rydberg atom by ultrashort pulse laser

Example of quantum phase/wavefunction sensitive physics

現実から夢へ: Aharonov-Bohm effect

Quantum dots, Nanotubes, ....

To understand the effect, consider the arrangement illustrated above. In this experiment, a wall with two narrow slits intercepts electrons from the source, and a detector on the other side registers the rate at which electrons arrive at a small region at a distance $a$ above the axis of symmetry. The rate is proportional to the probability that an individual electron will reach the region, which can be understood in terms of the interference of the wavefunctions $\psi_1 = C_1 e^{i k x}$ and $\psi_2 = C_2 e^{i n \varphi}$ passing through each slit. The phase difference $\delta = \phi_2 - \phi_1$ produces an interference pattern and, as shown above, the phase difference is given by

$$\delta = \frac{a}{\lambda} = \frac{4\pi}{\lambda}.$$ (1)

The interference of the waves at the detector depends on the phase difference, so

$$\delta = \phi_1 - \phi_2 = \psi_{1,2} - 0 = \frac{e}{h} \int_{\gamma_1}^{\gamma_2} A \cdot ds.$$ 

Possible to see if the Einstein-Podolsky-Rosen effect?
Conclusions

Relativistic engineering using intense lasers and relativistic coherence

• RE allows unprecedented regime of physical parameters: pulse length, intensity (of photons and other particles)…. Manifestation of the Duality Conjecture (Mourou)

• RE accesses Atto- and zepto- second regimes of (coherent) photon science, a paradigm change from $|\Psi|^2 \rightarrow \Psi$

• RE accesses extreme fields

• We may get more parameter regimes than explored so far
Centaurus A: cosmic wakefield linac?

Merci Beaucoup et à la Prochaine Fois!