Nonperturbative electron-positron pair creation in strong laser fields

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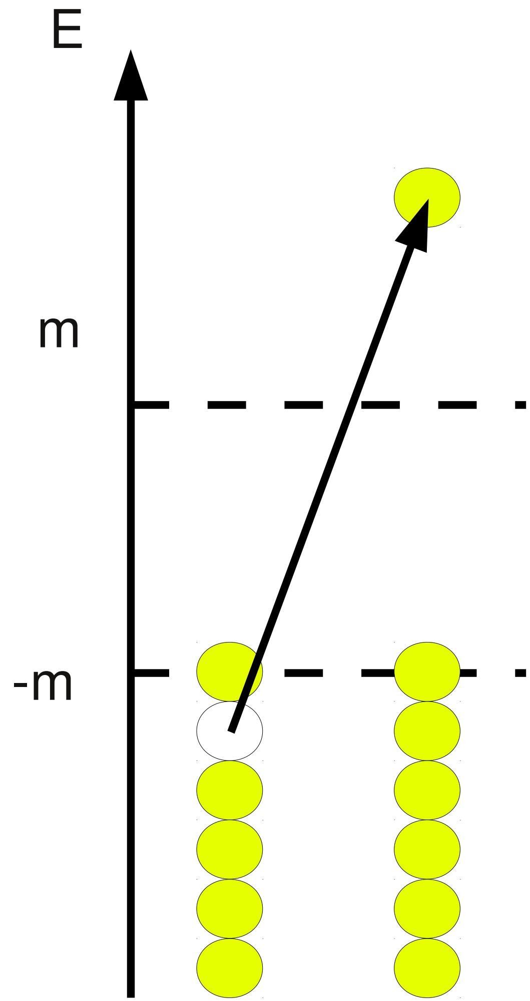
Introduction: e^+e^- pair creation in strong fields

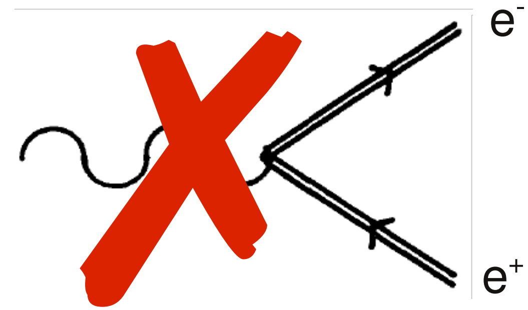


Schwinger field strength:

$$E_c = m^2/|e| \approx 10^{16} V/cm$$

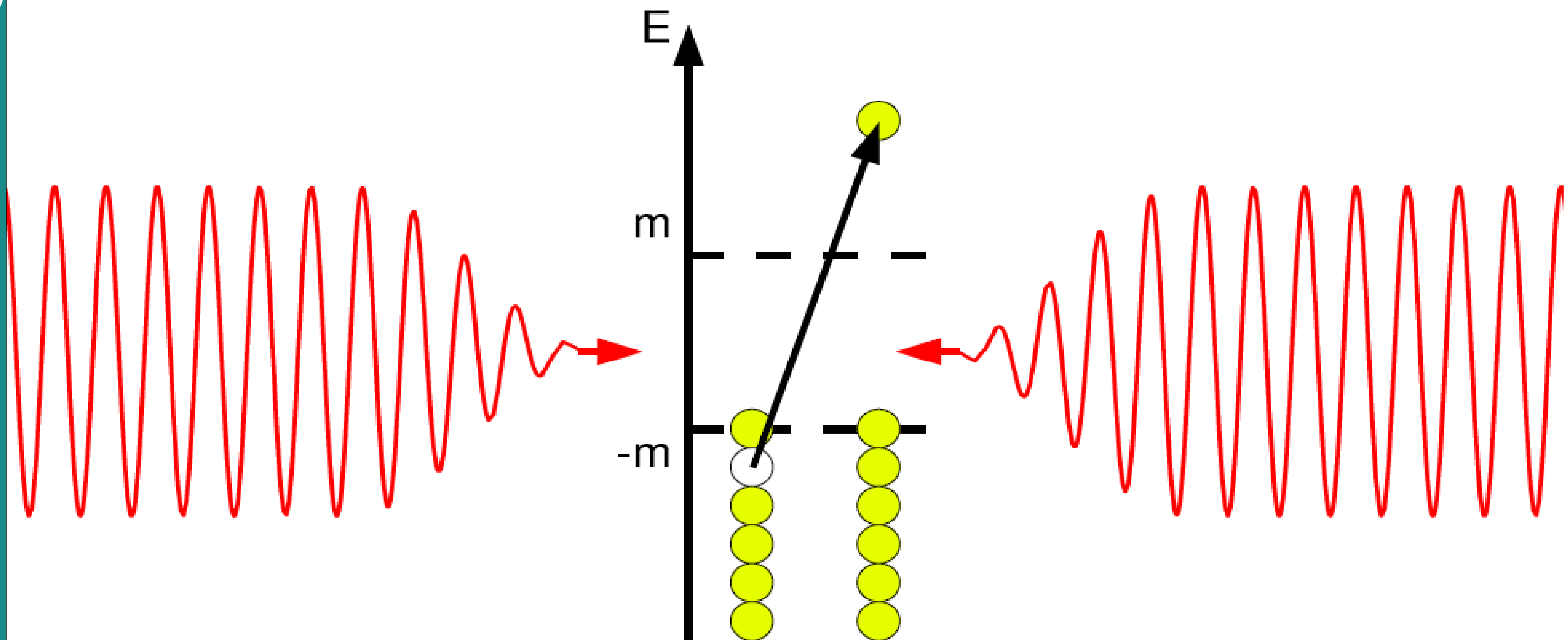
(natural units: $\hbar = c = 1$)

Note: A single plane laser wave cannot produce pairs.



This intuitive “Dirac sea” picture is in accordance with QFT approach of unstable vacuum in external fields.

pair creation in counter propagating laser fields



$$A = A_0[\sin(\omega t + kz) + \sin(\omega t - kz)]$$
$$= 2A_0 \sin(\omega t) \cos(kz)$$
$$\approx 2A_0 \sin(\omega t)$$

The typical length scale for pair creation is λ_C, which is much smaller than $\lambda = 2\pi/k$ at optical or infrared frequencies.

Quasiclassical estimate

$$\Psi_{\pm} \propto \exp\left[iS_{\pm}\right] \qquad \omega \ll m,\ E \ll E_c = m^2$$

classical action

$$S_{\pm} = \mathbf{qr} \mp \frac{1}{\omega}\int_{\eta_0}^{\eta} d\eta' \sqrt{m^2 + (\mathbf{q} - e\mathbf{A}(\eta'))^2)}$$

$$= \mathbf{qr} \mp q_0 t \pm S_p(\eta)$$

q_0 : averaged energy of the electron in the field

The transition amplitude is given by

$$\Psi_{-}^{\dagger}\Psi_{+} \propto \exp\left[i\left(S_{+} - S_{-}\right)\right]$$

$$= \sum_n \mathcal{C}_n \exp\left[i\left(n\omega - 2q_0\right)t\right]$$

n : emitted or absorbed laser photons

$$\Psi_{-}^{\dagger}\Psi_{+} \propto \sum_{n} \mathcal{C}_n \exp\left[i\left(n\omega - 2q_0\right)t\right]$$

This leads to the resonance condition $\omega = \dfrac{2q_0}{n}$

Note: The laser dressed energy enters the resonance condition!

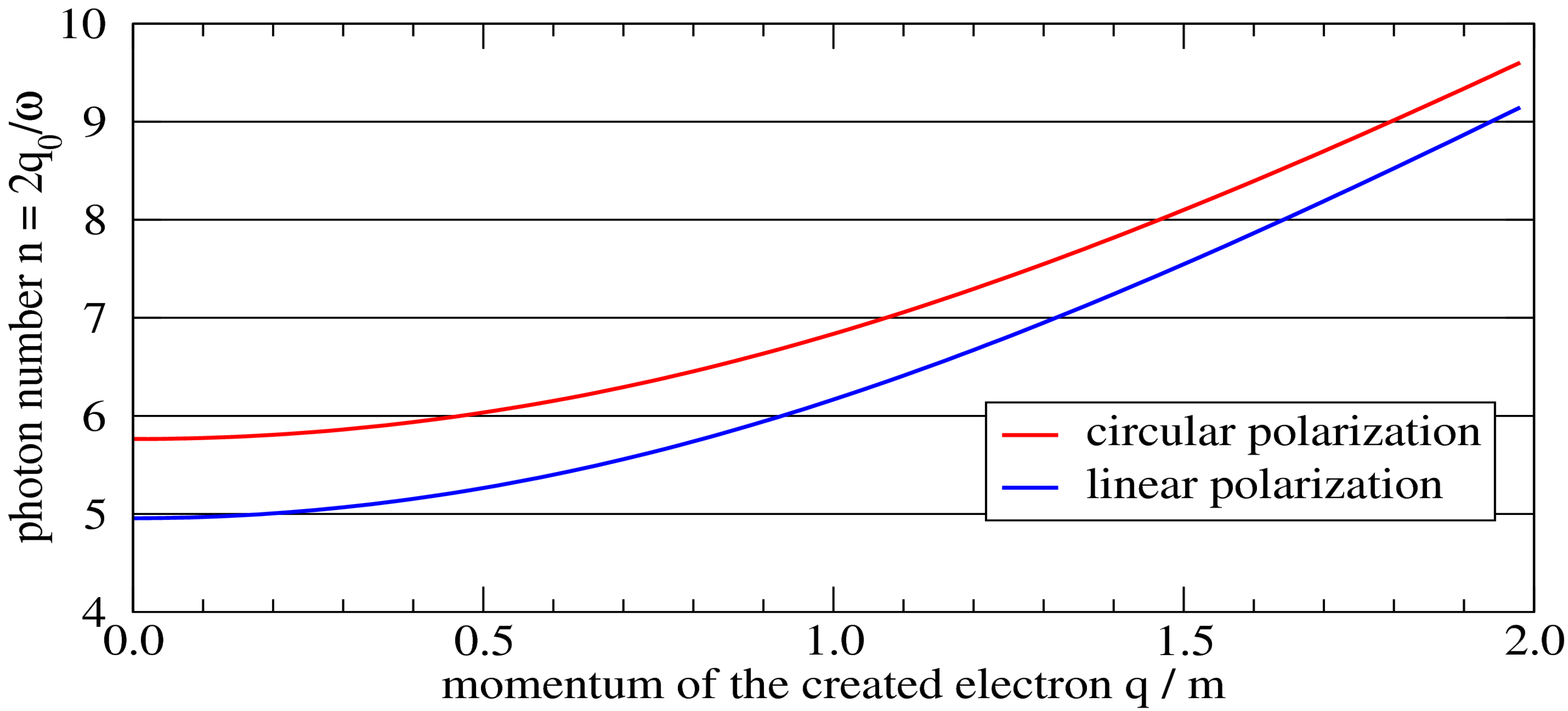
The probability of an n-photon process is given by

$$W_n \propto \left|\mathcal{C}_n\right|^2$$

resonance condition

scaled quasi-energy vs. the momentum of the created electron

$$n = \frac{2q_0}{\omega}$$



$\omega = 0.49m, \quad \xi = 1$ relativistic laser parameter $\xi = \frac{E}{m\omega}$

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different regimes of pair creation

(perturbative) $$W_n \propto \xi^{2n} \qquad \xi \ll 1$$

(intermediate) $$W \propto \exp\left[-\Gamma \frac{m}{\omega}\right] \qquad \xi = 1$$

$\Gamma \approx 3$ for circular polarization
$\Gamma \approx 2$ for linear polarization

(tunneling regime) $$W \propto \exp\left[-\pi \frac{m^2}{E}\right] \qquad \xi \gg 1$$

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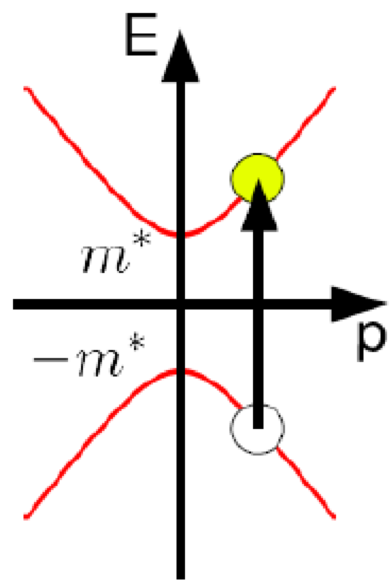
numerical treatment

From now on: $\xi = 1$

- Dirac equation
- Initial condition: negative-energy state
- Projection onto positive-energy states yield creation probability

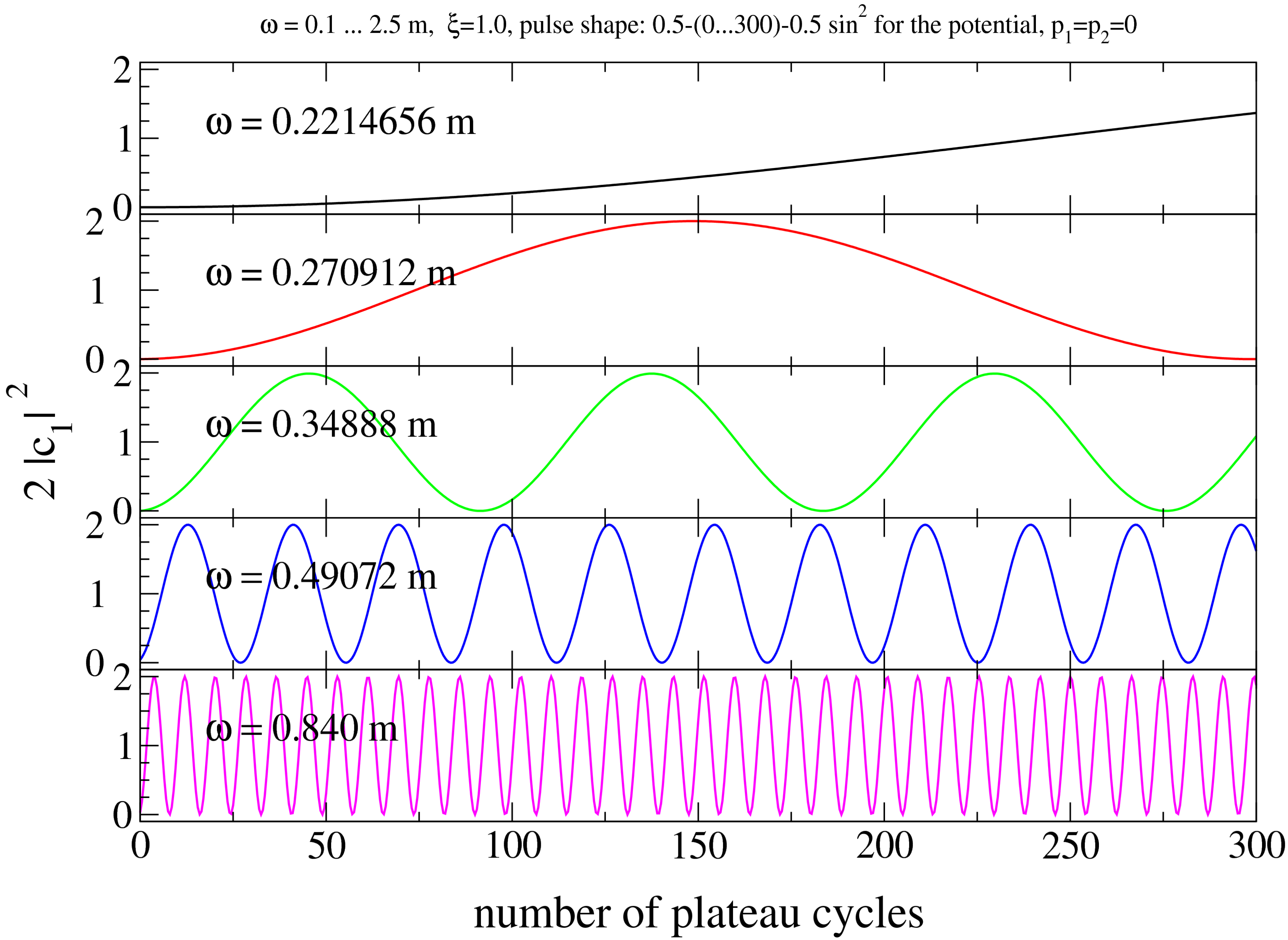
This leads to a Differential equation which is solved numerically



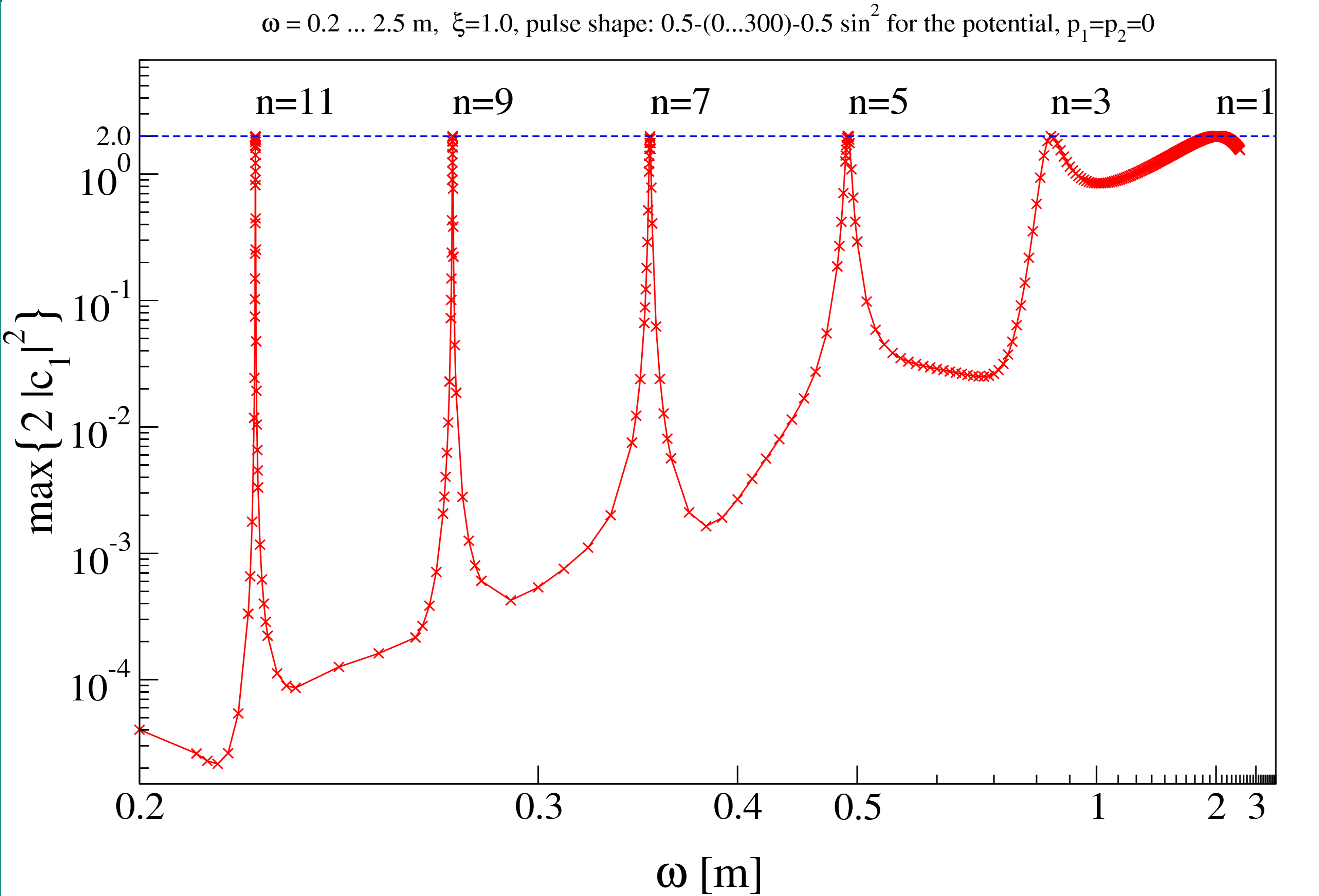
Pure two level system due to momentum conservation with resonance frequencies

$$\omega = \frac{2q_0}{n}$$

Rabi-oscillations at certain resonant frequencies



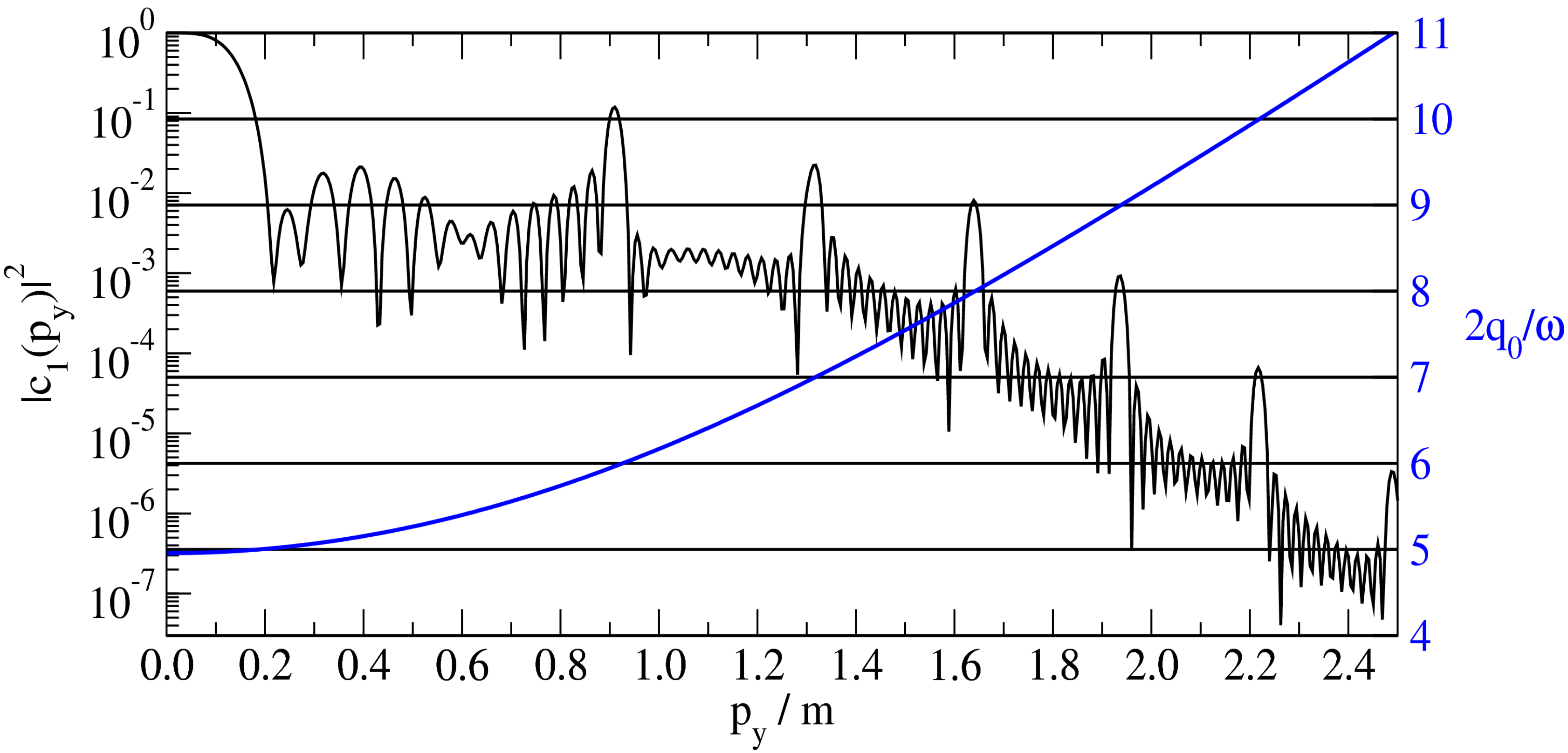
Resonances at p=0



• Introduction
• Quasi-classical cal.
• Numerics OEC
• Numerics CPL
• Summary

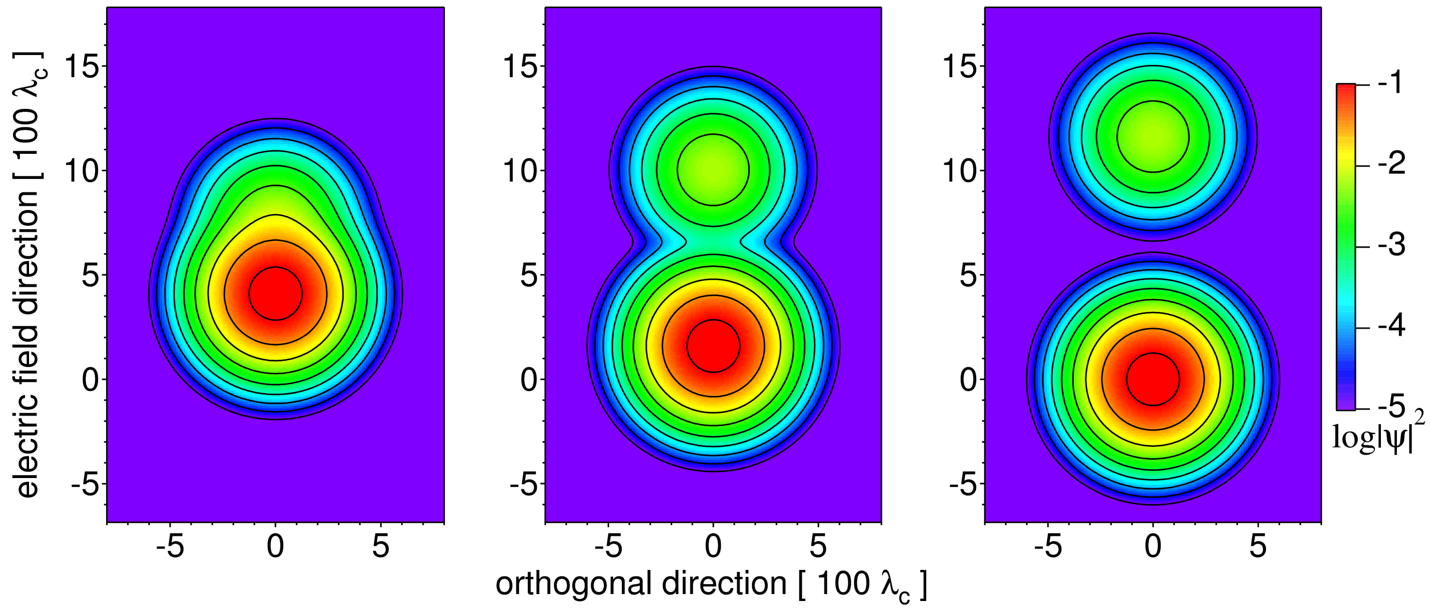
Towards total rates:

momentum distribution of the created electrons in polarization direction



fixed frequency $\omega = 0.49m$

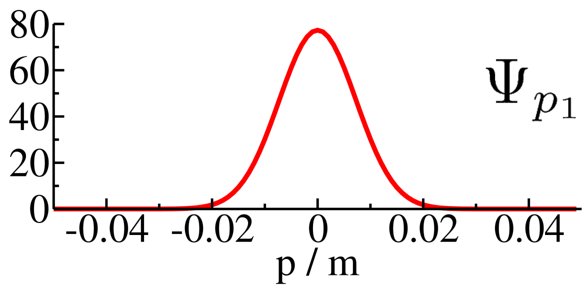
- Solving the Dirac-equation on a 2D-grid via the Split-Operator-Method.
- Initial State: negative energy electron
- Pair creation probability is given by projection onto all positive-energy states after the interaction.



$$\omega = \frac{m}{200}, \quad E_0 = E_c = \frac{m^2}{|e|}, \quad \xi = 200$$

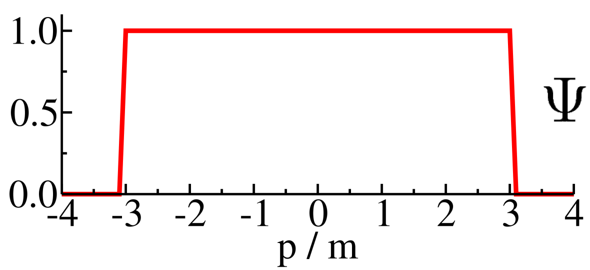
Numerical approach: running the code in a non-standard mode

Instead of using an initial negative-energy Gaussian wave-packet, representing one single electron in the Dirac-sea:



$$\Psi_{p_1,\pm} \approx \int d^3p\,\delta(p_z)\exp[-\left(\frac{\mathbf{p}-\mathbf{p}_1}{2\sigma}\right)^2]\Phi^{(-)}_{p,\pm}$$

We use a modified wave function, enabling us to do the integral over the initial momenta before the propagation.

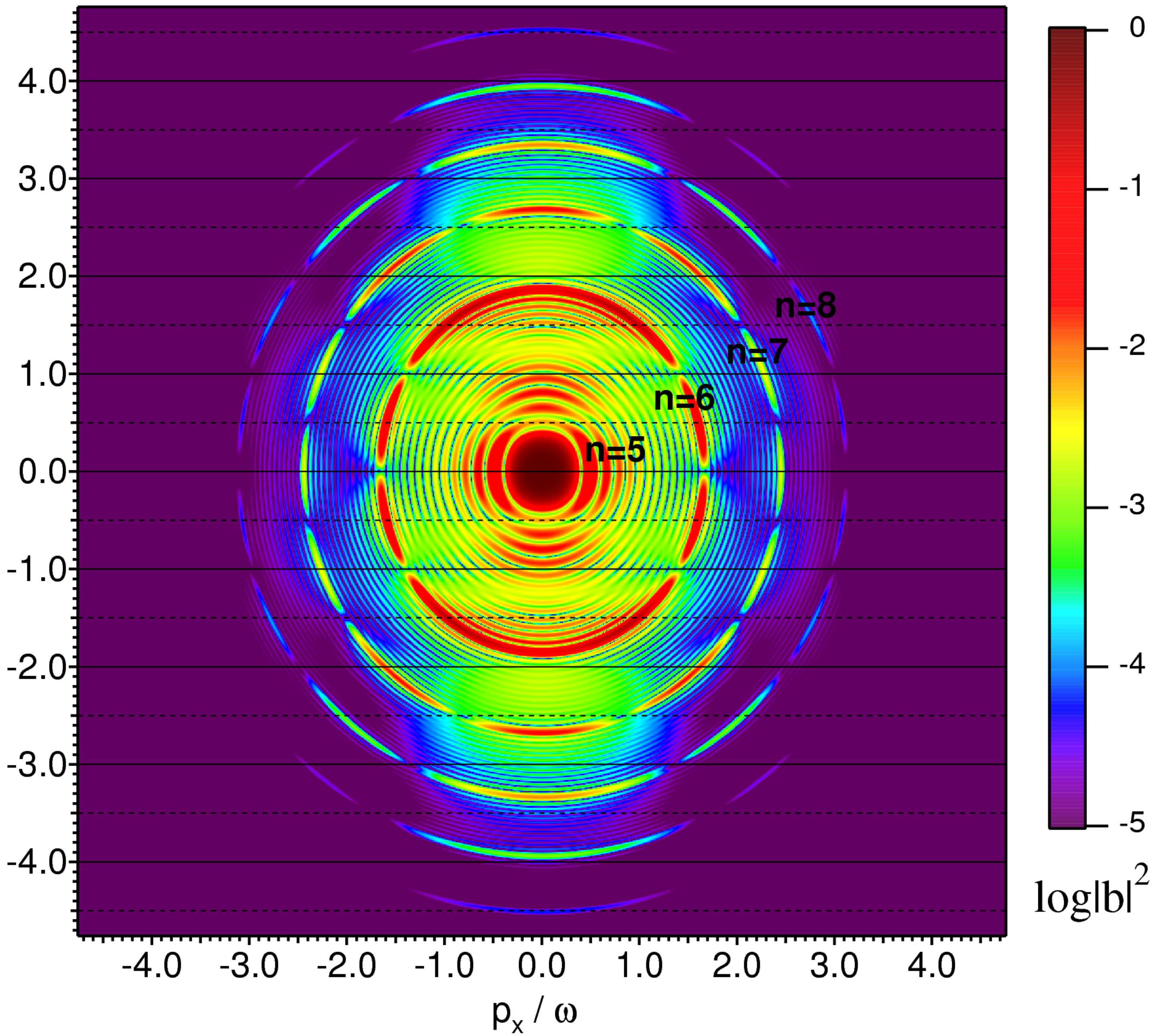


$$\Psi_{\Upsilon,\pm} \approx \int d^3p\,\delta(p_z)\,\theta(\Upsilon^2 - p_x^2 - p_y^2)\,\Phi^{(-)}_{p,\pm}$$

After the interaction, projection onto all free positive-energy states yields the momentum density of the created electron in just one calculation.

Momentum distribution of the created electrons: Fingerprints of the vacuum

$\omega = 0.49072$ m, corresponding to an n=5 photon resonance and a 0.5 | 13.0 | 0.5 pulse shape. Independent of the initial spin.



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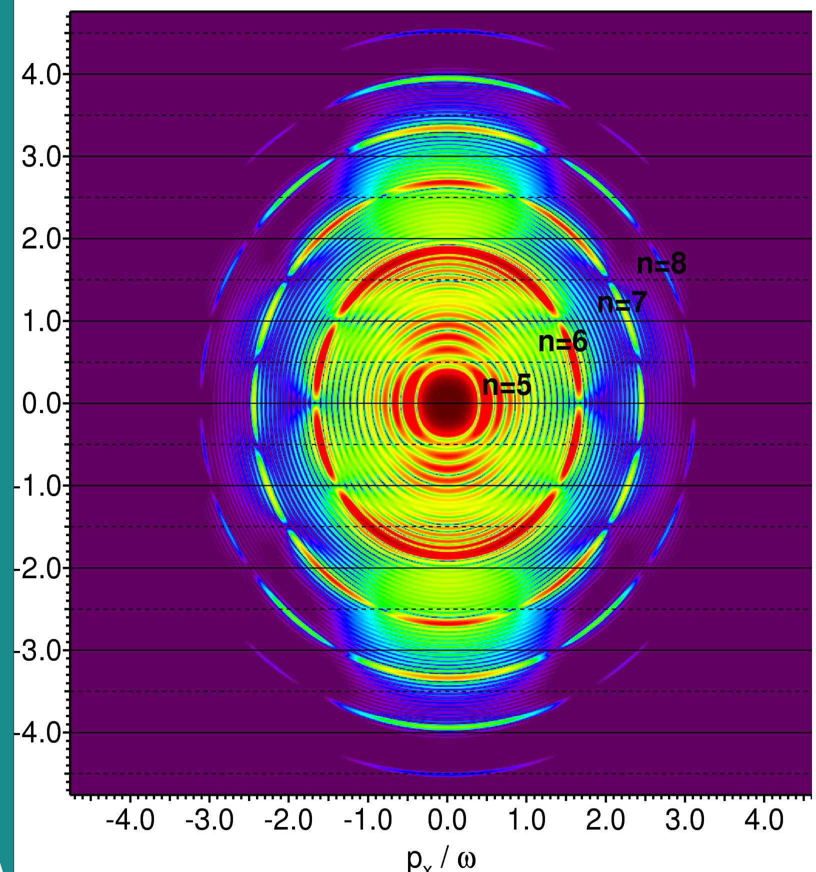
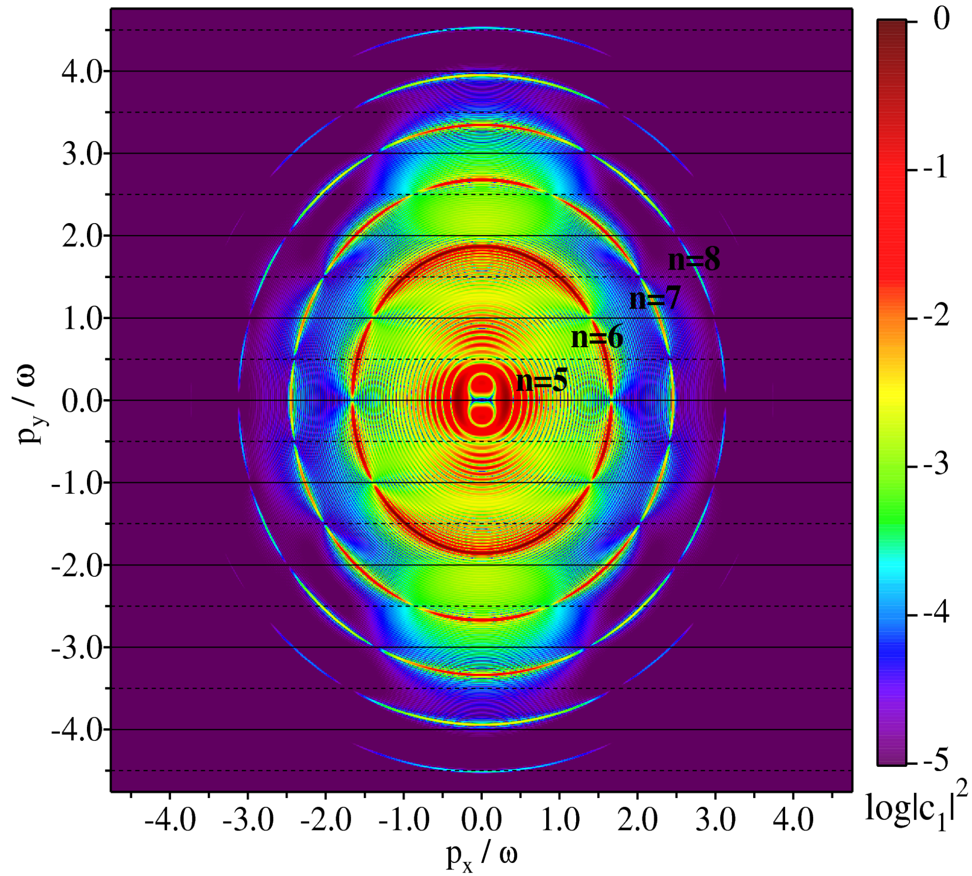
Influence of the Rabi-oscillation onto the momentum distribution

$\omega = 0.49072$ m, corresponding to an n=5 photon resonance for two different pulse lengths



0.5 - 13.5 - 0.5



0.5 - 27.0 - 0.5

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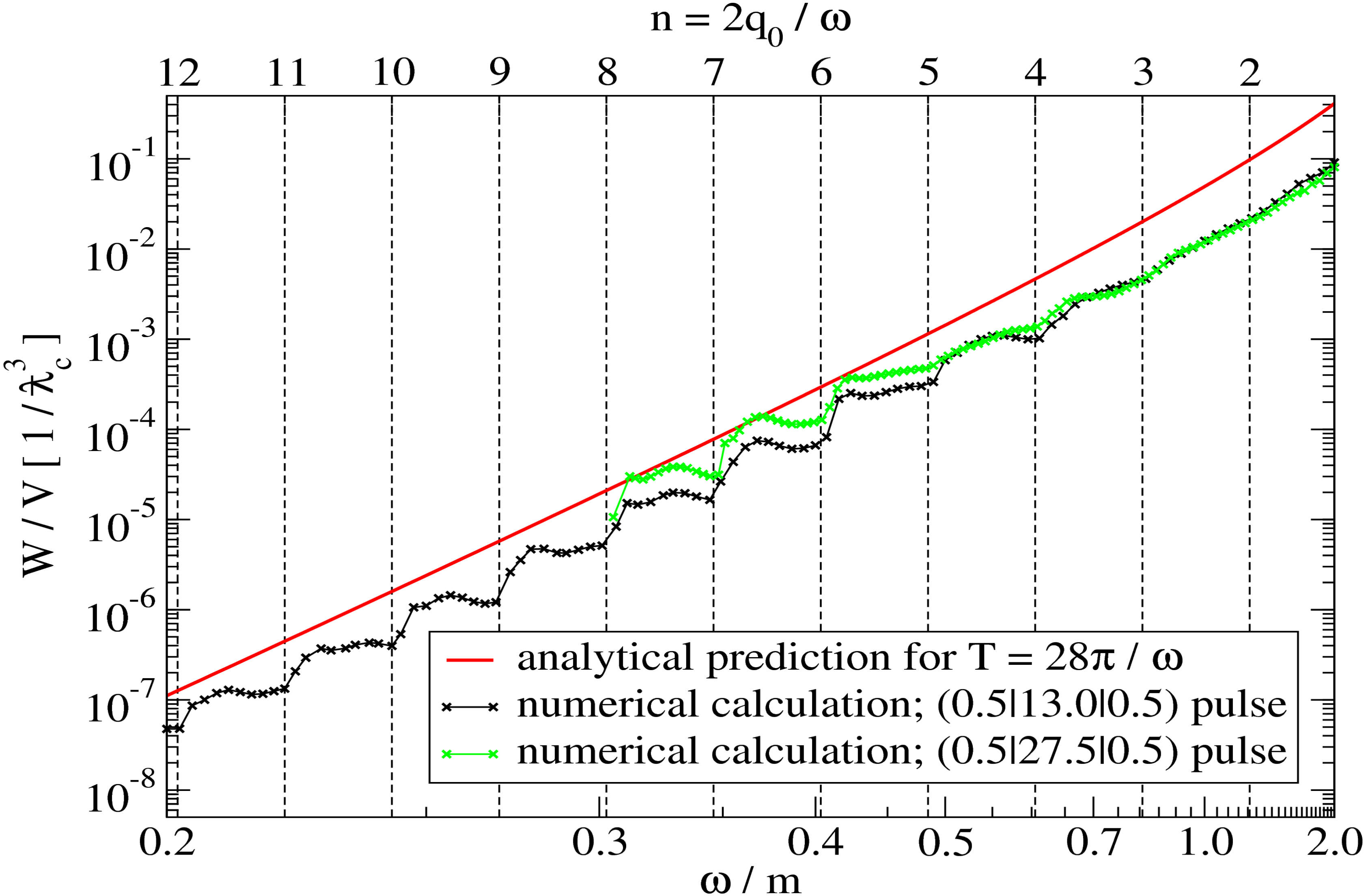
Integration of these fingerprints (over the momentum grid) yields the total pair creation probability per volume



e.g. for $\omega = 0.2m,\ V = (10\lambda)^3$ approximately one pair is produced!

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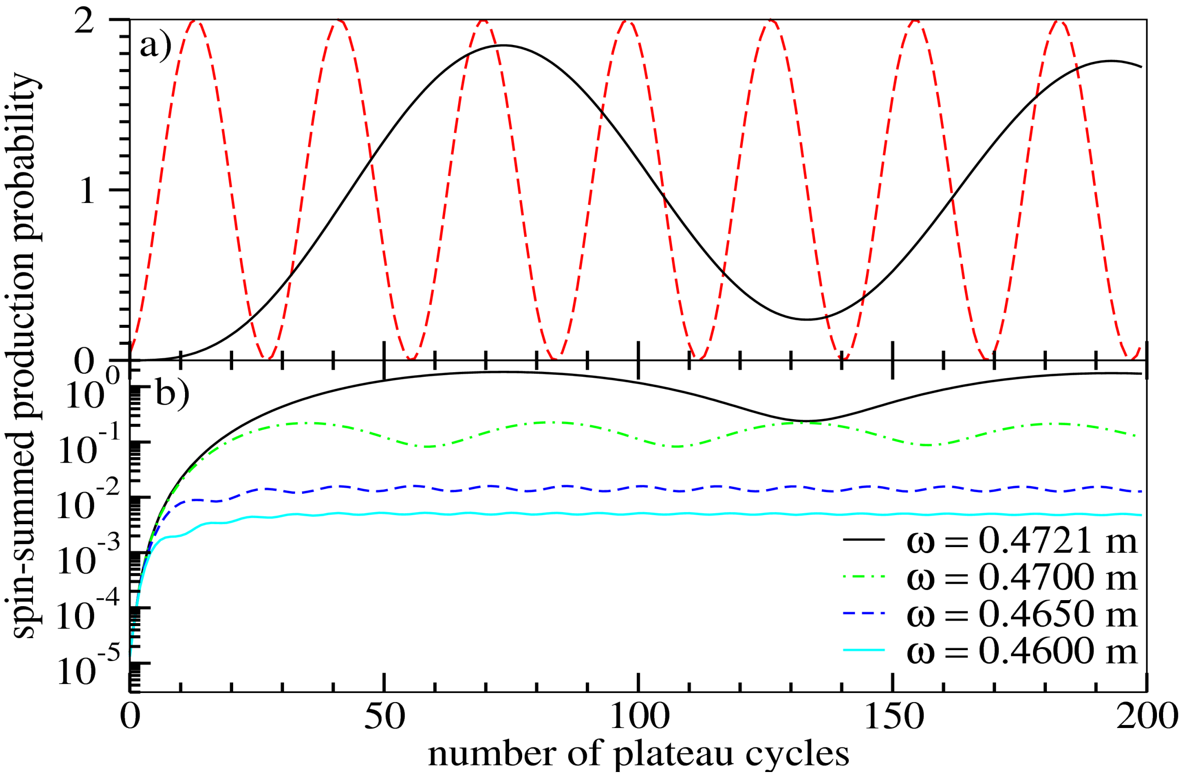
Including the Magnetic Field

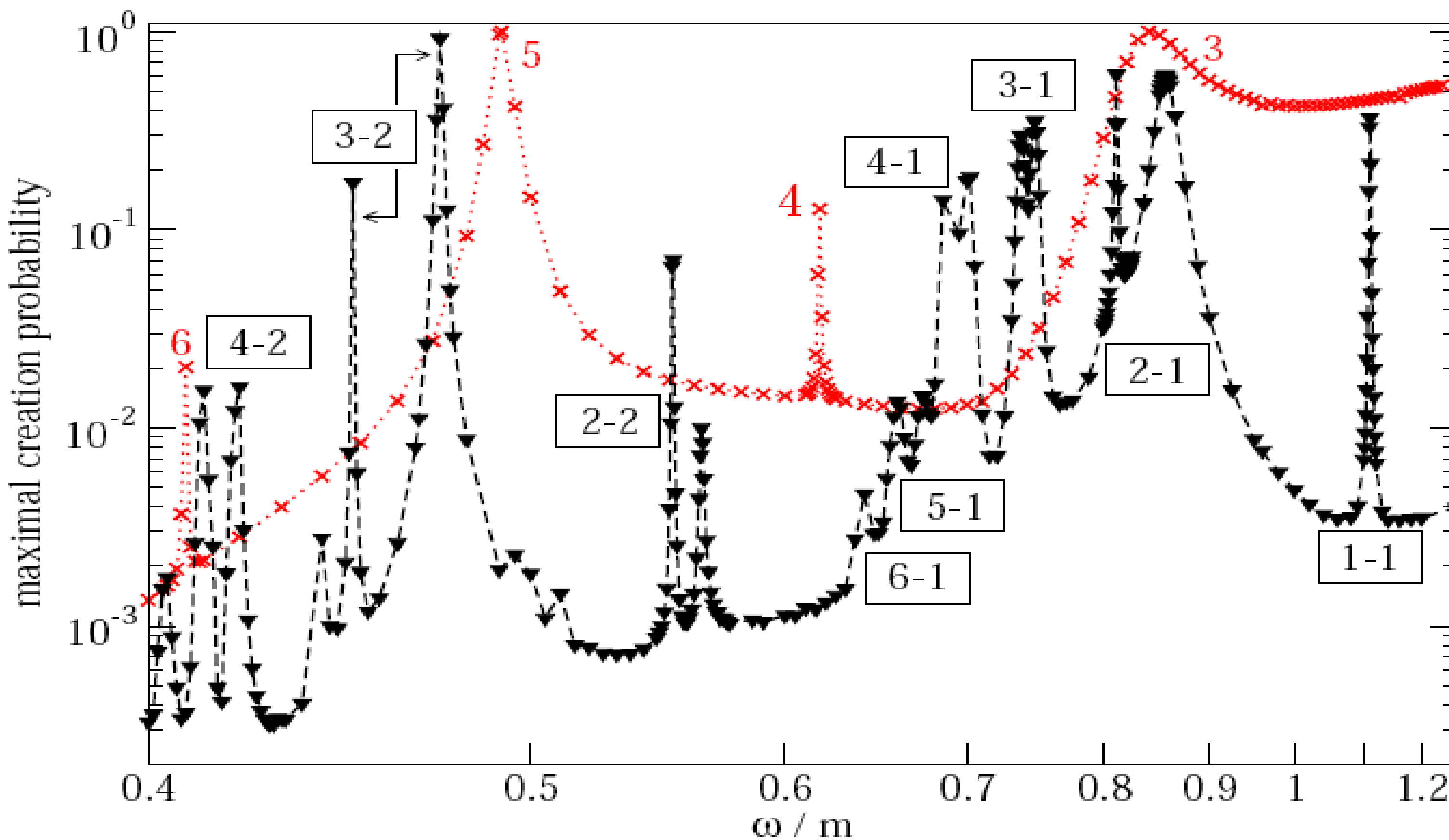
The process is also extensively discussed in view of XFEL devices, calling for (numerical) calculations including the space dependence.

The magnetic component of the laser field modifies the Rabi-oscillations.

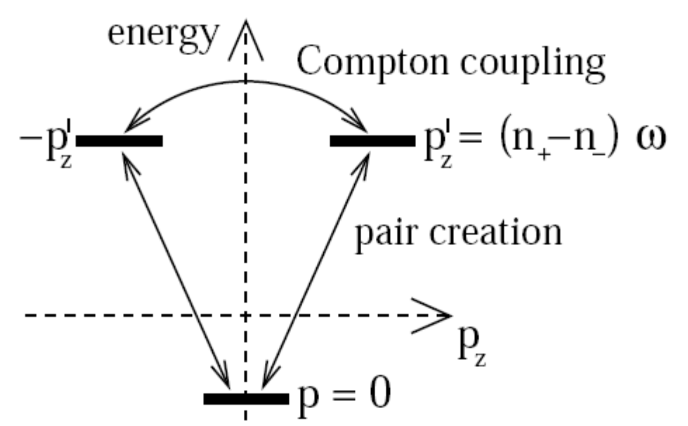


– – – Oscillating electric field for $\omega = 0.49072m$

Influence on the resonance structure



Due to the laser magnetic field, the photons carry longitudinal momentum. The resonances are shifted, multiplied, and split (Autler-Townes effect).



Ruf, Mocken, Müller, Hatsagortsyan & Keitel, PRL **102**, 080402 (2009)

Summary

- Quasiclassical treatment yields an order of magnitude estimation.
- Numerical analysis of the problem yields Rabi-oscilations, resonances, momentum distributions (fingerprints), and total rates.
- Pronounced magnetic field effects exist in pair creation in a standing wave of high frequency.